On the Derivation of Strong and Electro-weak Interactions in Functional Quantum Theory of the Nonlinear Spinor Field as a Lepton-hadron Model with Quark-confinement

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In preceding papers the nonlinear spinor field with Heisenberg's dipole regularization was interpreted as a lepton-quark system. In this paper it is demonstrated that this model is able to produce electro-weak as well as strong interactions. For the analytical derivation of these interactions, in Section 1 an improved calculation technique for scattering functionals is developed which corresponds to an interaction representation of relativistic quantum fields with inclusion of bound states, and which is suited for a comparison of its results with conventional field theories, in particular gauge theories. In Section 2 the projection technique from the spinor field into the lepton-quark representation is discussed in detail. The principle which allows the derivation of the various interactions consists in the calculation and incorporation of universal (local) bosons and non-universal (non-local) bosons occuring as bound states of the spinor field, resp., lepton-quark fields, into the scattering functional equation. This is performed for local bosons in Section 1 and for non-local bosons in Section 3. In Section 4 it is shown that a subsequent unitarization which corresponds to quark confinement leads to selection rules for lepton-baryon processes which qualitatively correspond to those of grand unification gauge theories. Numerical calculations will be given in subsequent papers.

Functional quantum theory is a new formulation of quantum theory and a new field theoretic calculational procedure which allows the treatment of quantized fields with positive metric as well as with indefinite metric beyond perturbation theory. It was developed by Stumpf and coworkers, cf. Stumpf [1]. In particular, it was devoted to the evaluation of a consistent calculational formalism for Heisenberg's nonlinear spinor field equation with dipole ghost regularization, cf. Heisenberg [2]. For the theoretical evaluation of this model by means of functional quantum theory and for a proper physical interpretation of the corresponding calculational results, the physical meaning of this model must be explained and this explanation mainly depends on the interpretation of the one-particle sector of the spinor field.

In the original version of Heisenberg, the physical one-particle states of the spinor field were assumed to represent nucleons, while the leptons were sometimes identified with the regularizing dipole ghosts, cf. Heisenberg [2], Dürr [3]. The reversed role of hadrons and leptons was first proposed by Saller [4] who interpreted the spinor field as a Weinberg model

and assumed the leptons to be the physical particles and the quarks to be the dipole ghost particles. Both versions, however, were only vaguely formulated and no hint was given with respect to an analytical evaluation of the idea. In particular, no attempt was made to establish separate dynamical laws for both kinds of particles with quark confinement and to derive the hierarchy of interactions in such a model. In a recent note Stumpf [5] proposed an analytical method for the derivation of a lepton-quark dynamics from the fundamental spinor field including quark confinement, and in a subsequent paper Stumpf [6] formulated the fundamental spinor field equation for two fermion generations and drew first conclusions from the model in the framework of functional quantum theory. In particular, it was demonstrated that the local (electro-weak) bosons, which arise as bound states of the spinor field, are universally coupled to leptons and quarks, and that the lowest order of electroweak fermion-fermion scattering formulae of gauge-coupling theories can be reproduced with the spinor field formalism, at which boson masses and coupling constants are calculable quantities. In this paper we introduce an improved scattering functional calculation technique and an improved lepton-quark projection formalism and by means of this method we discuss the

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derivation of strong and electro-weak interactions in the framework of functional quantum theory for the spinor field model. The derivation of the corresponding dynamical equations etc. in the following sections is strictly deductive. However, the formalism of functional quantum theory will not be repeated here. Rather we refer to Stumpf [1] (and preceding papers), Stumpf [5], and Stumpf [6], which will be denoted in the following by I, resp. II and III.

1. Calculation of Scattering Functionals

The derivation and the evidence of the various forces, resp. interactions, between elementary particles depend on the analysis of elementary particle, resp. particle scattering processes. Hence for the theoretical discussion, scattering states have to be calculated and analyzed. Concerning functional quantum theory of the nonlinear spinor field, all particles and all elementary particles have to be considered as bound states of the fundamental field. Therefore, in this type of theory scattering processes as a general principle have to be calculated only between bound states. On the other hand, for a comparison with the results of conventional coupling-field-calculations, which are mainly done in the interaction representation, a functional calculation technique for scattering between bound states is needed which easily reproduces the results of these theories. Hence it has to be expected that a functional interaction representation for bound states will solve both requirements.

In the development of functional quantum theory, various calculation techniques for scattering functionals have been proposed and discussed. In the recent discussion of such calculational techniques, the formation and separation of the self-energy of local fermions (pointlike physical particles) as a first step of the calculational procedure has played the main role. In contrast to this effort, the local and non-local bosons play the main role for an analysis of the forces and the comparison with conventional coupling theories. Hence it is appropriate to postpone the problem of the formation of local fermions as bound states of the spinor field and to start with a Lagrangian, resp. a spinor field equation, where the formation of the local fermions has already been incorporated. Such an equation corresponds to a spinorial version of the Froissart model, Froissart

[7], where a nonlinear spinor field interaction term has been added. Equations of this type where studied by Dürr [8], however, not with respect to the model and the functional techniques which we use here.

For a first discussion of the hierarchy of forces we do not need the complete model of III for two fermion generations. Rather we can restrict ourselves to the model of II with a single lepton-quark pair. The corresponding field equations read

$$[(-i\boldsymbol{\delta} + \mu)^2(-i\boldsymbol{\delta} + m)]_{\alpha\beta}\,\psi_\beta(x)$$
$$= V_{\alpha\beta\gamma\delta}\,\psi_\beta(x)\,\bar{\psi}_\gamma(x)\,\psi_\delta(x) \qquad (1.1)$$

with $\boldsymbol{\delta} := \partial_\mu \gamma^\mu$.

If $\boldsymbol{\psi}(x)$ and $\bar{\boldsymbol{\psi}}(x)$ are combined to give a superspinor $\boldsymbol{\Psi}(x)$, cf. III, then the corresponding state functionals are given by

$$|\,\mathfrak{T}(j,a)\rangle := \sum_{n=1}^{\infty}\int \langle 0|\,T\Psi_{\alpha_1}(x_1)\dots\Psi_{\alpha_n}(x_n)\,|a\rangle$$
$$\cdot\,|\,D_n(x_1\dots x_n)\rangle\,\mathrm{d}^4x_1\dots\mathrm{d}^4x_n \quad (1.2)$$

and satisfy a corresponding functional equation which can be derived from (1.1). For actual calculations instead of (1.2) the state functionals

$$|\,\mathfrak{F}(j,a)\rangle := \exp[-\tfrac{1}{2}\int j_\alpha(x_1)\,F_{\alpha\beta}(x_1 - x_2)$$
$$\cdot\,j_\beta(x_2)\,\mathrm{d}^4x_1\,\mathrm{d}^4x_2]\,|\,\mathfrak{T}(j,a)\rangle \quad (1.3)$$

are used, where $F_{\alpha\beta}(x)$ is assumed to be the propagator of the corresponding linear superspinor field. The functional equation for $|\,\mathfrak{F}\rangle$ then reads

$$[D_{\alpha\beta}(x)\,\partial_\beta(x) + \tilde{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]$$
$$\cdot\,|\,\mathfrak{F}(j,a)\rangle = 0 \qquad (1.4)$$

with

$$D_{\alpha\beta}(x) := [(-i\,\hat{G}^\nu\,\partial_\nu + \mu)^2(-i\,\hat{G}^\varrho\,\partial_\varrho + m)]_{\alpha\beta} \qquad (1.5)$$

and

$$d_\beta(x) := \partial_\beta(x) + \int F_{\beta\alpha}(x - x')\,j_\alpha(x')\,\mathrm{d}^4x', \qquad (1.6)$$

where $\hat{G}$ and $\hat{V}$ are the superspinor quantities corresponding to the ordinary Dirac matrices $G_\mu \equiv \gamma_\mu$ and the vertex V. For the solution of Eq. (1.4), Stumpf [9] has proposed a solution procedure, cf. also I. In this paper we will generalize and improve this method in order to obtain a functional interaction representation for bound states.

In order to derive appropriate scattering functional equations, we observe that due to the gauge-invariance of Eq. (1.1) with respect to phase changes

$\psi'(x) = \exp(ia)\,\psi(x)$, a conserved fermion number ϱ can be introduced which allows a unique definition of a ϱ-fermion sector. If scattering processes in the ϱ-fermion sector have to be considered, we assume that the application of $D(x_2)\,\partial(x_2)\ldots D(x_\varrho)\,\partial(x_\varrho)$ to Eq. (1.4) leads to an appropriate starting point for scattering calculations. We demonstrate for the case $\varrho = 2$ that this assumption can successfully be evaluated. For $\varrho > 2$ the treatment runs on similar patterns. In the two fermion sector we obtain by applying $D_{\varkappa\nu}(y)\,\partial_\nu(y)$ to (1.4) and repeated use of (1.4)

$$\{D_{\varkappa\nu}(y)\,\partial_\nu(y)\,D_{\alpha\beta}(x)\,\partial_\beta(x) + [D_{\varkappa\nu}(y)\,\partial_\nu(y),\,\hat{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]_+$$
$$+ \hat{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)\,\hat{V}_{\varkappa\nu\varepsilon\lambda}\,d_\nu(y)\,d_\varepsilon(y)\,d_\lambda(y)\}\,|\,\mathfrak{F}(j,a)\rangle = 0\,. \tag{1.7}$$

Putting $F(x-x') \equiv F_{xx'}$ etc. and observing $F_{xx} = 0$ we can directly evaluate the commutator in (1.7). This leads to

$$[D_{\varkappa\nu}(y)\,\partial_\nu(y),\,\hat{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]_+ = D_{\varkappa\nu}(y)\,\hat{V}_{\alpha\beta\gamma\delta}[F_{xy}^{\nu\beta}\,F_{xx'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,F_{x'''}^{\delta\delta'}\,j_{x'''}^{\delta'} - F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,F_{xy}^{\gamma\nu}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}$$
$$+ F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,F_{xy}^{\gamma\gamma'}\,j_{x''}^{\gamma'}\,F_{xy}^{\delta\nu} + F_{xy}^{\beta\nu}\,F_{xx'}^{\gamma\gamma'}\,j_{x''}^{\gamma'}\,\partial_x^\delta - F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,F_{\gamma\nu}^{xy}\,\partial_x^\delta + F_{\beta\nu}^{xy}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}\,\partial_x^\gamma - F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,F_{xy}^{\delta\nu}\,\partial_x^\gamma$$
$$+ F_{xy}^{\gamma\nu}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}\,\partial_x^\beta - F_{xy'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,F_{xy}^{\delta\nu}\,\partial_x^\beta + F_{xy}^{\beta\nu}\,\partial_x^\gamma\,\partial_x^\delta + F_{xy}^{\gamma\nu}\,\partial_x^\beta\,\partial_x^\delta + F_{xy}^{\delta\nu}\,\partial_x^\beta\,\partial_x^\gamma]\,, \tag{1.8}$$

where equal coordinates between F and j mean integration. For the further evaluation of (1.7) and (1.8) the relations

$$D_{\alpha\beta}(x)\,F_{\beta\gamma}(x-x') = A_{\varkappa\gamma}\,\delta(x-x')\,, \tag{1.9}$$

and

$$G_{\alpha\beta}(x-x')\,D_{\beta\gamma}(x') = \delta_{\alpha\gamma}\,\delta(x-x') \tag{1.10}$$

cf. Pouradjam [10], have to be observed. Due to $F_{\alpha\beta}(x) = -F_{\beta\alpha}(-x)$ we obtain for (1.8)

$$[D_{\varkappa\nu}(y)\,\partial_\nu(y),\hat{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]_+ = \hat{V}_{\alpha\beta\gamma\delta}[-A_{\varkappa\beta}\,\delta_{xy}\,F_{xx'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'} + F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,A_{\varkappa\gamma}\,\delta_{xy}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}$$
$$- F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,F_{xx'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,A_{\varkappa\delta}\,\delta_{xy} - A_{\varkappa\beta}\,\delta_{xy}\,F_{xx'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,\partial_x^\delta + F_{xx'}^{\beta\beta'}\,j_{x'}^{\beta'}\,A_{\varkappa\gamma}\,\delta_{xy}\,\partial_x^\delta - A_{\varkappa\beta}\,\delta_{xy}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}\,\partial_x^\gamma$$
$$+ F_{xx'}^{\beta\beta'}\,A_{\varkappa\delta}\,\delta_{xy}\,\partial_x^\gamma - A_{\varkappa\gamma}\,\delta_{xy}\,F_{xx'''}^{\delta\delta'}\,j_{x'''}^{\delta'}\,\partial_x^\beta + F_{xx'}^{\gamma\gamma'}\,j_{x'}^{\gamma'}\,A_{\varkappa\delta}\,\delta_{xy}\,\partial_x^\beta - A_{\varkappa\beta}\,\delta_{xy}\,\partial_x^\gamma\,\partial_x^\delta$$
$$- A_{\varkappa\gamma}\,\delta_{xy}\,\partial_x^\beta\,\partial_x^\delta - A_{\varkappa\delta}\,\delta_{xy}\,\partial_x^\beta\,\partial_x^\gamma]\,. \tag{1.11}$$

In analogy to the solution procedure given in I, also in the generalized version a separation of the operator of Eq. (1.7) into diagonal and non-diagonal terms is needed. This gives for (1.11) the decomposition

$$[D_{\varkappa\nu}(y)\,\partial_\nu(y),\,\hat{V}_{\alpha\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]_+ = -\hat{V}_{\alpha\beta\gamma\delta}[A_{\varkappa\beta}\,\partial_\gamma(x)\,\partial_\delta(x) + A_{\varkappa\gamma}\,\partial_\beta(x)\,\partial_\delta(x)$$
$$+ A_{\varkappa\delta}\,\partial_\beta(x)\,\partial_\gamma(x)]\,\delta(x-y) + N_{\alpha\varkappa}^1(x,y)\,, \tag{1.12}$$

where $N_1(x,y)$ contains the non-diagonal operators.

Since in our general discussion the special dependence on the algebraic degrees of freedom does not play an essential role, we shall use in the following a symbolic notation. In this notation we write (1.12) in the form

$$[\mathbf{D}(y)\,\partial(y),\,\hat{\mathbf{V}}\,d(x)\,d(x)\,d(x)]_+ = \mathbf{V}_1\,\partial(x)\,\partial(x)\,\delta(x-y) + \mathbf{N}_1(x,y)\,. \tag{1.13}$$

Concerning the third term in (1.7) we perform a normal ordering with respect to $j(x)$ and $\partial(x)$. Decomposing into diagonal terms and non-diagonal terms we obtain in our symbolic notation

$$\hat{\mathbf{V}}\,d(x)\,d(x)\,d(x)\,\hat{\mathbf{V}}\,d(y)\,d(y)\,d(y) = \mathbf{V}_2\,\mathbf{F}(x-y)\,\mathbf{F}(x-y)\,\partial(x)\,\partial(y)$$
$$+ \mathbf{V}_2'\,\mathbf{F}(x-y)\,\mathbf{F}(x-y)\,\partial(y)\,\partial(y) + \mathbf{N}_2(x,y)\,. \tag{1.14}$$

With (1.13) and (1.14) Eq. (1.7) takes the form

$$[\mathbf{D}(y)\,\mathbf{D}(x)\,\partial(y)\,\partial(x) + \mathbf{V}_1\,\delta(x-y)\,\partial(x)\,\partial(x) + \mathbf{V}_2'\,\mathbf{F}(x-y)\,\mathbf{F}(x-y)\,\partial(y)\,\partial(y)$$
$$+ \mathbf{V}_2\,\mathbf{F}(x-y)\,\mathbf{F}(x-y)\,\partial(x)\,\partial(y)]\,|\,\mathfrak{F}(j,a)\rangle = \mathbf{N}(x,y)\,|\,\mathfrak{F}(j,a)\rangle \tag{1.15}$$

with

$$\mathsf{N}(x,y) := -\,\mathsf{N}_1(x,y) - \mathsf{N}_2(x,y)\,. \tag{1.16}$$

We now apply $\mathbf{G}$ of (1.10) to (1.15) with respect to x and y and obtain with

$$|\,\mathfrak{F}(x,y)\rangle := \partial(x)\,\partial(y)\,|\,\mathfrak{F}(j,a)\rangle \tag{1.17}$$

for Eq. (1.15) the expression

$$
\begin{aligned}
|\,\mathfrak{F}(x,y)\rangle &+ \int \mathbf{G}(x-x')\,\mathbf{G}(y-x')\,\mathbf{V}_1\,|\,\mathfrak{F}(x'x')\rangle\,\mathrm{d}^4x' \\
&+ \int \mathbf{G}(x-x')\,\mathbf{G}(y-y')\,\mathbf{V}_2'\,\mathbf{F}(x'-y')\,\mathbf{F}(x'-y')\,|\,\mathfrak{F}(y'y')\rangle\,\mathrm{d}^4x'\,\mathrm{d}^4y' \\
&+ \int \mathbf{G}(x-x')\,\mathbf{G}(y-y')\,\mathbf{V}_2\,\mathbf{F}(x'-y')\,\mathbf{F}(x'-y')\,|\,\mathfrak{F}(x'y')\rangle\,\mathrm{d}^4x'\,\mathrm{d}^4y' \\
&= \int \mathbf{G}(x-x')\,\mathbf{G}(y-y')\,\mathsf{N}(x'y')\,|\,\mathfrak{F}(j,a)\rangle\,\mathrm{d}^4x'\,\mathrm{d}^4y' + |\,\mathfrak{F}(x'y')\rangle_0\,,
\end{aligned}
\tag{1.18}
$$

where $|\,\mathfrak{F}(x',y')\rangle_0$ is a solution of the equation

$$\mathbf{D}(y)\,\mathbf{D}(x)\,\partial(y)\,\partial(x)\,|\,\mathfrak{F}(j)\rangle_0 = 0\,. \tag{1.19}$$

Fourier-Transformation of (1.18) and suppression of the tilde for the Fourier-Transformed quantities lead to the equation

$$
\begin{aligned}
|\,\mathfrak{F}(s_1 s_2)\rangle &+ \int \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\delta(s_1+s_2-p_1-p_2)\,\mathbf{V}_1\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 \\
&+ \int \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathbf{V}_2'\,\mathbf{F}(-s_1-r)\,\mathbf{F}(r)\,\mathrm{d}^4r\,\delta(s_1+s_2-p_1-p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 \\
&+ \int \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathbf{V}_2\,\mathbf{F}(-s_1-r+p_1)\,\mathbf{F}(r)\,\mathrm{d}^4r\,\delta(s_1+s_2-p_1-p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 \\
&= \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathsf{N}(s_1 s_2)\,|\,\mathfrak{F}(j,a)\rangle + |\,\mathfrak{F}(s_1 s_2)\rangle_0\,.
\end{aligned}
\tag{1.20}
$$

In III an equation for the two fermion amplitude $\varphi(s_1 s_2)$ was derived. By comparison of (1.20) with Eqs. (2.17) and (2.18) of III it can be seen, that the first three terms of (1.20) represent the local boson operator, but now applied to the total state functional. However, the formation of a resolvent with respect to local bosons is independent whether it is applied to a simple amplitude or to a state functional. Hence the solution procedure of III, Section 3, can be taken over and applied to Eq. (1.20). We rewrite Eq. (1.20) in the form

$$|\,\mathfrak{F}(s_1 s_2)\rangle + \mathbf{W}(s_1 s_2)\int \delta(s_1+s_2-p_1-p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 = \mathfrak{R}(s_1 s_2) \tag{1.21}$$

using the definitions

$$\mathbf{W}(s_1 s_2) := \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)[\mathbf{V}_1 + \mathbf{V}_2'\,\mathbf{M}(s_1)]\,, \qquad \mathbf{M}(s_1) := \int \mathbf{F}(-s_1-r)\,\mathbf{F}(r)\,\mathrm{d}^4r\,, \tag{1.22},\ (1.23)$$

$$
\begin{aligned}
\mathsf{R}(s_1 s_2) := &-\int \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathbf{V}_2\,\mathbf{M}(s_1-p_1)\,\delta(s_1+s_2-p_1-p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 \\
&+ \mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathsf{N}(s_1 s_2)\,|\,\mathfrak{F}(j,a)\rangle + |\,\mathfrak{F}(s_1 s_2)\rangle_0\,.
\end{aligned}
\tag{1.24}
$$

Then according to III (3.11) we obtain

$$|\,\mathfrak{F}(s_1 s_2)\rangle = \mathbf{W}(s_1 s_2)\sum_\alpha \mathbf{D}_\alpha(s_1+s_2)\,g_\alpha^{-1}(s_1+s_2)\,\mathsf{R}(s_1+s_2) + \mathsf{R}(s_1 s_2) \tag{1.25}$$

with

$$\mathsf{R}(s_1+s_2) := \int \delta(s_1+s_2-p_1-p_2)\,\mathsf{R}(p_1 p_2)\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 \tag{1.26}$$

and

$$\mathbf{D}_\alpha(s) := \boldsymbol{\chi}^\alpha \otimes \boldsymbol{\chi}^\alpha(s^2 - m_\alpha^2)^{-1}\,, \tag{1.27}$$

where $\mathbf{D}_\alpha(s)$ are the local boson propagators, cf. III, Section 3. The only difference of (1.27) compared with III is the appearance of an additional selfenergy term $\mathbf{M}(s_1)$ in the boson resolvent which does not change its qualitative behaviour. If Eq. (1.25) is explicitly evaluated, it takes the form

$$
\begin{aligned}
|\,\mathfrak{F}(s_1 s_2)\rangle = &\int \mathbf{K}(s_1 s_2,\, p_1 p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 + \mathbf{W}(s_1 s_2)\sum_\alpha \mathbf{D}_\alpha(s_1+s_2)\,g_\alpha^{-1}(s_1+s_2) \\
&\cdot \int \delta(s_1+s_2-p_1-p_2)\,\mathbf{G}(-s_1)\,\mathbf{G}(-s_2)\,\mathsf{N}(s_1 s_2)\,|\,\mathfrak{F}(j,a)\rangle\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2
\end{aligned}
$$

$$+ \mathbf{W}(s_1 s_2) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2) g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2) \, | \, \mathfrak{F}(p_1 p_2)\rangle_0 \, \mathrm{d}^4 p_1 \, \mathrm{d}^4 p_2$$

$$+ \mathbf{G}(-s_1) \mathbf{G}(-s_2) \mathbf{N}(s_1 s_2) \, | \, \mathfrak{F}(j, a)\rangle + | \, \mathfrak{F}(s_1 s_2)\rangle_0 \tag{1.28}$$

with

$$\mathbf{K}(s_1 s_2, p_1 p_2) := \mathbf{G}(-s_1) \mathbf{G}(-s_2) \mathbf{V}_2 \mathbf{M}(s_1 - p_1) \, \delta(s_1 + s_2 - p_1 - p_2)$$

$$+ \mathbf{W}(s_1 s_2) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2) g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2)$$

$$\cdot \, \delta(p_1' + p_2' - p_1 - p_2) \mathbf{G}(-p_1') \mathbf{G}(-p_2') \mathbf{V}_2 \mathbf{M}(p_1' - p_1) \mathrm{d}^4 p_1' \, \mathrm{d}^4 p_2'. \tag{1.29}$$

In this form the exchange of local bosons, which are universally coupled to all pointlike fermions is directly incorporated in the functional interaction term. I. e. if now an interaction representation for the fermions is used, bound local boson states are included in the mutual fermion-fermion interaction.

2. Functional Lepton-quark Representations

As was emphasized in II, the field operators of the spinor field $\psi(x)$ contain an unobservable mixture of lepton- and quark-field operators. Hence it remains the problem of the formulation of separate dynamical laws for both kinds of particles. We relate the solution of this problem to the definition of the hierarchy of interactions. We observe that as long as the interactions of the fermions with universal (local) bosons are considered, there is no need for a theoretical separation of both kinds of particles in the dynamical law. If, however, interactions are considered which are not mediated by universal bosons, the dynamical laws for both kinds of particles must be formulated separately. From this we conclude the principle:

i) The boson states which mediate universal interactions must be incorporated into the dynamical laws before projection into the lepton-quark representation.

ii) The boson states which mediate non-universal interactions must be incorporated into the dynamical laws after projection into the lepton-quark representation.

That this principle leads to a distinction, for instance, of electro-weak and strong interactions, will be demonstrated in the following. Concerning i) we already have incorporated the universal bosons into the dynamical law, namely the functional equation for the scattering functional in the first section. As we assume that by this procedure all universal bosons have been considered, we now have to perform the next step, namely the projection of the dynamical law, i.e. the scattering functional equation, into the lepton-quark representation. In order

to do this we will improve the projection method given in II. We observe that the complete theoretical formalism applied to the spinor field is formulated in an interaction representation with respect to the fermions; i.e. the fermion propagator is assumed to be identical with that of the free lepton-ghost field, cf. I, which should be justified by a subsequent selfconsistency calculation. It is therefore natural to derive the projection technique by means of this interaction representation, too. In this case we have to study the linear spinor field equations only. The free field equation for the spinor field reads in our approach (i.e. with the exclusion of the selfenergy and selfformation problem of fermions)

$$(-i\boldsymbol{\delta} + \mu)^2 (-i\boldsymbol{\delta} + m) \boldsymbol{\psi}(x) = 0. \tag{2.1}$$

We now introduce auxiliary spinorial fields $\boldsymbol{A}(x)$ and $\boldsymbol{B}(x)$ and replace Eq. (2.1) by the set of equivalent equations

$$(-i\boldsymbol{\delta} + \mu) \boldsymbol{A}(x) = 0,$$
$$(-i\boldsymbol{\delta} + m) \boldsymbol{B}(x) = \boldsymbol{A}(x), \tag{2.2}$$
$$(-i\boldsymbol{\delta} + \mu) \boldsymbol{\psi}(x) = \boldsymbol{B}(x).$$

In order to obtain a lepton-quark representation we perform the transformation $\boldsymbol{A}(x) = \boldsymbol{A}(x)$ and

$$\boldsymbol{B}(x) = \boldsymbol{l}(x) + \alpha \boldsymbol{A}(x),$$
$$\boldsymbol{\psi}(x) = -\alpha \boldsymbol{l}(x) + \boldsymbol{q}(x) \tag{2.3}$$

with $\alpha := (m - \mu)^{-1}$ which leads to the set of equivalent equations

$$(-i\boldsymbol{\delta} + \mu) \boldsymbol{A}(x) = 0,$$
$$(-i\boldsymbol{\delta} + m) \boldsymbol{l}(x) = 0, \tag{2.4}$$
$$(-i\boldsymbol{\delta} + \mu)^l \boldsymbol{q}(x) = 0.$$

The fields $\boldsymbol{A}(x), \boldsymbol{B}(x), \boldsymbol{\psi}(x)$ can be quantized according to the rules of the conventional Lagrangian for-

malism, and the corresponding anticommutators can be calculated. The following anticommutators have been derived by Pouradjam [10]

$$[A(x), \overline{\Psi}(x')]_+ = [\Psi(x), \overline{A}(x')]_+$$
$$= -iS(x - x', \mu), \qquad (2.5)$$

$$[B(x), \overline{\Psi}(x')]_+ = [\Psi(x), \overline{B}(x')]_+$$
$$= -i\alpha[S(x - x', \mu) - S(x - x', m)],$$

$$[B(x), \overline{B}(x')]_+ = -iS(x - x', m), \qquad (2.5)$$

$$[\Psi(x), \overline{\Psi}(x')]_+ = -i\alpha^2 S(x - x', m)$$
$$+ i\alpha\left(\alpha + \frac{\partial}{\partial\mu}\right)S(x - x', \mu)$$

while all other anticommutators vanish. By means of these formulae the anticommutators of the transformed fields $l(x)$ and $q(x)$ can directly be derived. They read

$$[l(x), \bar{l}(x')]_+ = -iS(x - x', m),$$

$$[q(x), \bar{q}(x')]_+ = i\alpha\left(\alpha + \frac{\partial}{\partial\mu}\right)S(x - x', \mu),$$

$$[q(x), \bar{l}(x')]_+ = [l(x), \bar{q}(x')]_+ = 0, \qquad (2.6)$$

and in addition the relation holds

$$[\Psi(x), \overline{\Psi}(x')]_+ = \alpha^2[l(x), \bar{l}(x')]_+$$
$$+ [q(x), \bar{q}(x')]_+. \qquad (2.7)$$

From (2.6) it can be seen that $l(x)$ propagates as a real particle field, while $q(x)$ exhibits pure ghost propagation with no interference between the $l(x)$ and the $q(x)$ field.

For the transition into the lepton-quark representation the projection properties of $\Psi(x)$ with respect to these fields are needed. To derive these properties we perform a decomposition of $\Psi(x)$ into completely free fields $A_0(x)$ and $B_0(x)$ with the field equations

$$(-i\boldsymbol{\delta} + \mu)A_0(x) = 0,$$
$$(-i\boldsymbol{\delta} + m)B_0(x) = 0. \qquad (2.8)$$

According to Pouradjam [10] we obtain

$$\Psi(x) = A_0(x) - \alpha B_0(x)$$
$$- \mu^{-1}\alpha(\lambda - x^\nu \partial_\nu)A_0(x) \qquad (2.9)$$

as the general solution for $\Psi(x)$ of the system (2.2). With (2.3) and (2.4) the projection of $\Psi(x)$ into the lepton representation is given by

$$(-i\boldsymbol{\delta} + \mu)^2 \Psi(x) = -(1/\alpha)l(x) \qquad (2.10)$$

while the projection into the quark-representation gives by means of (2.3), (2.4), (2.8), (2.9) the expression

$$(-i\boldsymbol{\delta} + m)\Psi(x) = -\alpha q(x) - \mu A_0(x). \qquad (2.11)$$

By comparison of (2.3) and (2.9) the quark field has to be identified with

$$q(x) = A_0(x) - \mu^{-1}\alpha(\lambda - x^\nu \partial_\nu)A_0(x). \qquad (2.12)$$

The inconvenient term $\mu A_0(x)$ on the right-hand side of (2.11) can be removed if it is observed that the equations for $\Psi(x)$ and $q(x)$ are gauge invariant with respect to the addition of $A_0(x)$-fields. I. e. with $\Psi(x)$ and $q(x)$ also $\Psi'(x) = \Psi(x) + a_1 A_0(x)$ and $q'(x) = q(x) + a_2 A_0(x)$ are solutions of the equations

$$(-i\boldsymbol{\delta} + \mu)^2(-i\boldsymbol{\delta} + m)\Psi(x) = 0,$$
$$(-i\boldsymbol{\delta} + \mu)^2 q(x) = 0. \qquad (2.13)$$

If an arbitrary solution $\Psi(x)$ is given which leads to (2.11), then by means of the gauge transformation

$$\Psi'(x) = \Psi(x) + (\alpha - 1/\alpha)^{-1}\mu A_0(x),$$
$$q'(x) = q(x) + (\alpha - 1/\alpha)^{-1}\mu A_0(x). \qquad (2.14)$$

it can be achieved that the following equations simultaneously hold

$$\Psi'(x) = -\alpha l(x) + q'(x) \qquad (2.15)$$

and

$$(-i\boldsymbol{\delta} + \mu)^2 \Psi'(x) = -1/\alpha\, l(x),$$
$$(-i\boldsymbol{\delta} + m)\Psi'(x) = -\alpha q'(x). \qquad (2.16)$$

The formulae (8), (10), (11) of I can directly be recalculated to give (2.15) and (2.16). However, in contrast to I, no reference with respect to corresponding states of the linear field is required. In the following we shall use (2.15), (2.16) as a convenient starting point for the transformation into the lepton-quark representation.

First of all we have to rewrite the state functionals (1.2), resp. (1.3), into the lepton-quark representation. To achieve this, we observe the identity

$$\Psi(x) = \langle D_0 | \partial(x) \int \Psi(x')j(x')\,\mathrm{d}^4x' | D_0\rangle \qquad (2.17)$$

which has to hold in both representations. This can be secured by replacing in the lepton-quark representation the expression (2.17) by

$$\Psi(x) = \langle D_0 | [\partial_l(x) + \partial_q(x)]$$
$$\cdot \int [-\alpha l(x')j_l(x') + q(x')j_q(x')]$$
$$\cdot \mathrm{d}^4x' | D_0\rangle \qquad (2.18)$$

with the conditions

$$[\partial_l(x)\,j_l(x')]_+ = [\partial_q(x),\,j_q(x')]_+$$
$$= \delta(x - x') \qquad (2.19)$$

while all other anticommutators have to vanish. Then $\psi(x)$ is reproduced also in this representation.

Denoting for brevity

$$-\alpha\,l(x) =: \psi_1(x); \quad q(x) = \psi_2(x)$$

the general state functional (1.2) in the superspinor lepton-quark representation reads

$$|\mathfrak{T}(j,a)\rangle = \sum_{n=1}^{\infty} \int \langle 0|\, T\Psi_{i_1\alpha_1}(x_1)\dots\Psi_{i_n\alpha_n}(x_n)$$
$$|a\rangle\,|D_n(\overset{i_1\alpha_1}{x_1}\dots\overset{i_n\alpha_n}{x_n})\rangle\,\mathrm{d}^4x_1\dots\mathrm{d}^4x_n \qquad (2.20)$$

while the transformed $|\mathfrak{F}\rangle$ takes the form

$$|\mathfrak{F}(j,a)\rangle = \exp\big[-\tfrac{1}{2}\int j_{i_1\alpha}(x_1)\,F_{\alpha\beta}^{i_1 i_2}(x_1 - x_2)$$
$$\cdot j_{i_2\beta}(x_2)\,\mathrm{d}^4x_1\,\mathrm{d}^4x_2\big]\,|\mathfrak{T}(j,a)\rangle. \qquad (2.21)$$

The propagator $F_{\alpha\beta}^{i_1 i_2}(x)$ follows from the anticommutator by an appropriate change of the integration path in the momentum representation, but has the same Fouriertransform. Hence from (2.6) and (2.7) it follows that the propagator is diagonal with respect to the lepton-quark indices, i.e.

$$F_{\alpha\beta}^{i_1 i_2}(x) = \delta^{i_1 i_2} F_{\alpha\beta}(x, i_1). \qquad (2.22)$$

These formulae are sufficient to perform the transition into the lepton-quark representation in the functional scattering equation, as this transition will be performed by the simultaneous application of the projection operations (2.16) to the functional equation and by the substitution of the new field variables (2.15).

This procedure is especially convenient with respect to the separation of the local bosons in the original spinor field representation. It is, however, not necessary to proceed in this way, since the transformation (2.15), (2.16) remains meaningful beyond the interaction representation. If the nonlinear Eq. (1.1) is transformed by (2.15), (2.16), the nonlinear equations

$$(-i\,\delta + m)\,l(x) = -\alpha\,\mathbf{V}[-\alpha\,l(x) + q(x)]$$
$$\cdot [-\alpha\,\bar{l}(x) + \bar{q}(x)]$$
$$\cdot [-\alpha\,l(x) + q(x)],$$

$$(-i\,\delta + \mu)^2\,q(x) = -\frac{1}{\alpha}\,\mathbf{V}[-\alpha\,l(x) + q(x)]$$
$$\cdot [-\alpha\,\bar{l}(x) + \bar{q}(x)]$$
$$\cdot [-\alpha\,l(x) + q(x)] \qquad (2.23)$$

result and these equations can be used for the derivation of the corresponding functional equation in the usual way. By application of the transformation (2.15), (2.16) to the nonlinear Eq. (1.1), the mapping of the original spinor field onto the lepton-quark fields produces a unique map for the corresponding field equations, but the map is not bi-unique. As the original spinor field has no physical meaning, this loss of a one-to-one correspondence is no drawback for the theory. On the other hand, a map which secures the one-to-one correspondence of the corresponding equations can easily be achieved by considering the equations

$$(-i\,\delta + \mu)\,A(x) = \mathbf{V}\,\psi(x)\,\bar{\psi}(x)\,\psi(x),$$
$$(-i\,\delta + m)\,B(x) = A(x), \qquad (2.24)$$
$$(-i\,\delta + \mu)\,\psi(x) = B(x),$$

and applying the transformation (2.3). We do not discuss such a bi-unique map in more detail, since we prefer to work with the transformation (2.15), (2.16) and the corresponding Equations (2.23). In addition, we shall directly transform the functional equation by (2.15), (2.16) and we will not fall back upon (2.23) for the transition to the lepton-quark representation.

3. Universal and Non-universal Interactions

According to our program, in this section we perform the incorporation of the non-universal bosons into the scattering functional equation leading thus to non-universal interactions. In order to achieve this we must explicitly introduce the lepton-quark representation of the scattering functional equation. Defining the projectors

$$\mathbf{P}(1) := -\alpha^2(-i\,\delta + \mu)^2,$$
$$\mathbf{P}(\lambda) := -\alpha^{-1}(-i\,\delta + m), \qquad (3.1)$$

we apply these quantities to Eq. (1.28) and obtain by means of (2.15) and (2.16)

$$\mathbf{P}(i_1)\,\mathbf{P}(i_2)\,|\mathfrak{F}(s_1 s_2)\rangle = |\mathfrak{F}(\overset{i_1\ i_2}{s_1 s_2})\rangle$$
$$= \int \mathbf{K}(\overset{i_1\ i_2}{s_1 s_2}, p_1 p_2)[\,|\mathfrak{F}(\overset{1\ 1}{p_1 p_2})\rangle + |\mathfrak{F}(\overset{1\ 2}{p_1 p_2})\rangle + |\mathfrak{F}(\overset{2\ 1}{p_1 p_2})\rangle + |\mathfrak{F}(\overset{2\ 2}{p_1 p_2})\rangle]\,\mathrm{d}^4p_1\,\mathrm{d}^4p_2$$

$$+ \mathbf{W}(\overset{i_1}{s_1}\overset{i_2}{s_2}) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2)\, \mathbf{G}(-p_1)\, \mathbf{G}(-p_2)\, \mathbf{N}(p_1 p_2)$$

$$\cdot \,|\,\mathfrak{F}(j, a)\rangle\, \mathrm{d}^4p_1\, \mathrm{d}^4p_2$$

$$+ \mathbf{W}(\overset{i_1}{s_1}\overset{i_2}{s_2}) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2)\,|\,\mathfrak{F}(p_1 p_2)\rangle_0\, \mathrm{d}^4p_1\, \mathrm{d}^4p_2$$

$$+ \mathbf{G}(-\overset{i_1}{s_1})\, \mathbf{G}(-\overset{i_2}{s_2})\, \mathbf{N}(s_1 s_2)\,|\,\mathfrak{F}(j, a)\rangle + |\,\mathfrak{F}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle_0\,. \tag{3.2}$$

We now solve the equations

$$|\,\mathfrak{F}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle = \int \mathbf{K}(\overset{i_1}{s_1}\overset{i_2}{s_2}, p_1 p_2)\,|\,\mathfrak{F}(\overset{i_1}{p_1}\overset{i_2}{p_2})\rangle\, \mathrm{d}^4p_1\, \mathrm{d}^4p_2 + |\,\mathfrak{J}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle \tag{3.3}$$

by explicit construction of the corresponding resolvent. Owing to the properties of the kernels and the inhomogeneous terms which follow from (1.28) and (1.29), we may separate the center of mass momentum by introduction of center of mass coordinates s_c, s_r, resp. p_c, p_r, cf. III (2.19), and obtain from (3.3) the equation

$$|\,\mathfrak{F}(\overset{i_1}{s_c}\overset{i_2}{s_r})\rangle = \int \hat{\mathbf{K}}(\overset{i_1}{s_c}\overset{i_2}{s_r}, s_c\, p_r)\,|\,\mathfrak{F}(\overset{i_1}{s_c}, \overset{i_2}{p_r})\rangle\, \mathrm{d}^4p_r + |\,\mathfrak{J}(\overset{i_1}{s_c}\overset{i_2}{s_r})\rangle\,. \tag{3.4}$$

This equation can formally be solved and gives

$$|\,\mathfrak{F}(\overset{i_1}{s_c}\overset{i_2}{s_r})\rangle = |\,\mathfrak{J}(\overset{i_1}{s_c}\overset{i_2}{s_r})\rangle + \int \frac{\hat{\mathbf{R}}(\overset{i_1}{s_c}\overset{i_2}{s_r}s_c\, p_r)}{D(s_c)}\, \mathfrak{J}(\overset{i_1}{s_c}\overset{i_2}{p_r})\rangle\, \mathrm{d}^4p_r\,, \tag{3.5}$$

where $\hat{\mathbf{R}}(\overset{i_1}{s_c}\overset{i_2}{s_r}, s_c\, p_r)\, D^{-1}(s_c)$ is the general resolvent operator and $D(s_c)$ the secular polynom, which determines by its zeros the eigenvalues of the corresponding homogeneous equation. Equation (3.5) can be rewritten in the form

$$|\,\mathfrak{F}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle = |\,\mathfrak{J}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle + \int \delta(s_1 + s_2 - p_1 - p_2)\frac{\hat{\mathbf{R}}(\overset{i_1}{s_1}\overset{i_2}{s_2}\, p_1\, p_2)}{D(s_1 + s_2)}\,|\,\mathfrak{J}(\overset{i_1}{p_1}\overset{i_2}{p_2})\rangle\, \mathrm{d}^4p_1\, \mathrm{d}^4p_2\,. \tag{3.6}$$

A more detailed inspection of the homogeneous equation shows that it describes non-local boson states. These states depend on the special lepton-quark combination, since the kernel of (3.3), resp. (3.4), depends on i_1 and i_2. Hence in general we have to expect $(l \otimes l)$-bosons, $(l \otimes q)$-bosons and $(q \otimes q)$-bosons, which have different masses and eigenstates and which are non-local, as the resolvent operator is a non-local quantity. In analogy with the treatment of local bosons we may, however, still express the resolvent by a sum of non-local boson propagators, i.e. we may write

$$\frac{\hat{\mathbf{R}}(\overset{i_1}{s_1}\overset{i_2}{s_2}, p_1\, p_2)}{D(s_1 + s_2)} = \sum_\beta \frac{\hat{\mathbf{R}}_\beta(\overset{i_1}{s_1}\overset{i_2}{s_2}, p_1\, p_2)}{(s_c{}^2 - m_\beta{}^2)}\, g_\beta^{-1}(s_c) =: \sum_\beta \mathbf{B}_\beta(\overset{i_1}{s_1}\overset{i_2}{s_2}, p_1\, p_2)\, g_\beta^{-1}(s_1 + s_2)\,. \tag{3.7}$$

Then Eq. (3.6) goes over into

$$|\,\mathfrak{F}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle = |\,\mathfrak{J}(\overset{i_1}{s_1}\overset{i_2}{s_2})\rangle$$

$$+ \int \delta(s_1 + s_2 - p_1 - p_2) \sum_\beta \mathbf{B}_\beta(\overset{i_1}{s_1}\overset{i_2}{s_2}, p_1\, p_2)\, g_\beta^{-1}(s_1 + s_2)\,|\,\mathfrak{J}(\overset{i_1}{p_1}\overset{i_2}{p_2})\rangle\, \mathrm{d}^4p_1\, \mathrm{d}^4p_2\,. \tag{3.8}$$

This resolvent operator can now be applied to Equations (3.2). We define the matrix

$$C_{j_1 j_2}^{i_1 i_2} := (1 - \delta_{j_1}^{i_1} \delta_{j_2}^{i_2}) \tag{3.9}$$

and obtain from (3.2) by application of (3.7) the equations

$$
\begin{aligned}
|\, \mathfrak{F}(\overset{i_1\ i_2}{s_1 s_2})\rangle = &\sum_{j_1 j_2} \int \mathbf{K}(\overset{i_1\ i_2}{s_1 s_2},\, p_1\, p_2)\, C^{i_1 i_2}_{j_1 j_2} |\, \mathfrak{F}(\overset{j_1\ j_2}{p_1 p_2})\rangle\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{W}(\overset{i_1\ i_2}{s_1 s_2}) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2) \\
&\quad \cdot \mathbf{G}(-p_1)\, \mathbf{G}(-p_2)\, \mathbf{N}(p_1 p_2) |\, \mathfrak{F}(j, a)\rangle\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{W}(\overset{i_1\ i_2}{s_1 s_2}) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2) \int \delta(s_1 + s_2 - p_1 - p_2) |\, \mathfrak{F}(p_1 p_2)\rangle_0\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{G}(-\overset{i_1}{s_1})\, \mathbf{G}(-\overset{i_2}{s_2})\, \mathbf{N}(s_1 s_2) |\, \mathfrak{F}(j, a)\rangle + |\, \mathfrak{F}(\overset{i_1\ i_2}{s_1 s_2})\rangle_0 + \int \delta(s_1 + s_2 - q_1 - q_2) \\
&\quad \cdot \sum_\beta \mathbf{B}_\beta(\overset{i_1\ i_2}{s_1 s_2},\, q_1 q_2)\, g_\alpha^{-1}(s_1 + s_2) \Big\{ \sum_{j_1 j_2} \int \mathbf{K}(\overset{i_1 i_2}{q_1 q_2},\, p_1\, p_2)\, C^{i_1 i_2}_{j_1 j_2} |\, \mathfrak{F}(\overset{j_1\ j_2}{p_1 p_2})\rangle\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{W}(\overset{i_1\ i_2}{q_1 q_2}) \sum_\alpha \mathbf{D}_\alpha(q_1 + q_2)\, g_\alpha^{-1}(q_1 + q_2) \int \delta(q_1 + q_2 - p_1 - p_2) \\
&\quad \cdot \mathbf{G}(-p_1)\, \mathbf{G}(-p_2)\, \mathbf{N}(p_1 p_2) |\, \mathfrak{F}(j, a)\rangle\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{W}(\overset{i_1\ i_2}{q_1 q_2}) \sum_\alpha \mathbf{D}_\alpha(q_1 + q_2)\, g_\alpha^{-1}(q_1 + q_2) \int \delta(q_1 + q_2 - p_1 - p_2) |\, \mathfrak{F}(p_1 p_2)\rangle_0\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\
&+ \mathbf{G}(-\overset{i_1}{q_1})\, \mathbf{G}(-\overset{i_2}{q_2})\, \mathbf{N}(q_1 q_2) |\, \mathfrak{F}(j, a)\rangle + |\, \mathfrak{F}(\overset{i_1\ i_2}{q_1 q_2})\rangle_0 \Big\}\, \mathrm{d}^4 q_1\, \mathrm{d}^4 q_2 \,. \quad (3.10)
\end{aligned}
$$

Obviously the symmetry of the interactions between the lepton-sector and the quark-sector has been destroyed by the non-local bosons, since their properties depend on the special representation. Hence the non-local bosons mediate interactions which are non-universal, i.e. depend on the special sector under consideration. The meaning of these non-universal interactions, namely their interpretation as strong interactions, will be discussed in the next section. In this section we will conclude our formal analytical investigation of the scattering functional equation by the derivation of a solution procedure which corresponds to an interaction representation with inclusion of bound states. We write Eq. (3.10) in the general form

$$
|\, \mathfrak{F}(\overset{i_1\ i_2}{s_1 s_2})\rangle = \sum_{j_1 j_2} \int \mathbf{L}(\overset{i_1\ i_2}{s_1 s_2},\, \overset{j_1\ j_2}{p_1 p_2}) |\, \mathfrak{F}(\overset{j_1\ j_2}{p_1 p_2})\rangle\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 + \sum_{j_1 j_2} \int \mathbf{L}'(\overset{i_1\ i_2}{s_1 s_2},\, \overset{j_1\ j_2}{p_1 p_2}) |\, \mathfrak{F}(\overset{j_1\ j_2}{p_1 p_2})\rangle_0\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \,. \quad (3.11)
$$

The state functional $|\, \mathfrak{F}(p_1 p_2)\rangle$ may be replaced by

$$
|\, \mathfrak{F}(\overset{j_1\ j_2}{p_1 p_2})\rangle \equiv \partial_{j_1}(p_1)\, \partial_{j_2}(p_2) |\, \mathfrak{F}(j, a)\rangle \,. \quad (3.12)
$$

Multiplication of Eq. (3.11) by $j_{i_1}(s_1) j_{i_2}(s_2)$ and integration over s_1 and s_2 as well as summation over i_1 and i_2 then gives with

$$
|\, \mathfrak{F}(j, a)\rangle = \sum_{i_1 i_2} \int j_{i_1}(s_1)\, j_{i_2}(s_2) |\, \mathfrak{F}(\overset{i_1\ i_2}{s_1 s_2})\rangle\, \mathrm{d}^4 s_1\, \mathrm{d}^4 s_2 \quad (3.13)
$$

the equation

$$
\begin{aligned}
|\, \mathfrak{F}(j, a)\rangle = &\sum_{\substack{i_2 i_1 \\ j_1 j_2}} \int j_{i_1}(s_1)\, j_{i_2}(s_2)\, \mathbf{L}(\overset{i_1\ i_2}{s_1 s_2},\, \overset{j_1\ j_2}{p_1 p_2})\, \partial_{j_1}(p_1)\, \partial_{j_2}(p_2)\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2\, \mathrm{d}^4 s_1\, \mathrm{d}^4 s_2 |\, \mathfrak{F}(j, a)\rangle \\
&+ \sum_{\substack{i_1 i_2 \\ j_1 j_2}} \int j_{i_1}(s_1)\, j_{i_2}(s_2)\, \mathbf{L}'(\overset{i_1\ i_2}{s_1 s_2},\, \overset{j_1\ j_2}{p_1 p_2})\, \partial_{j_1}(p_1)\, \partial_{j_2}(p_2)\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2\, \mathrm{d}^4 s_1\, \mathrm{d}^4 s_2 |\, \mathfrak{F}(j, a)\rangle_0 \quad (3.14)
\end{aligned}
$$

which can formally be written

$$
|\, \mathfrak{F}(j, a)\rangle = \mathbf{L} |\, \mathfrak{F}(j, a)\rangle + \mathbf{L}' |\, \mathfrak{F}(j, a)\rangle_0 \,. \quad (3.15)
$$

The solution which corresponds to an interaction representation is given by

$$
\begin{aligned}
|\, \mathfrak{F}(j, a)\rangle = &\sum_{n=1}^\infty \mathbf{L}^n \mathbf{L}' |\, \mathfrak{F}(j, a)\rangle_0 \\
&+ \mathbf{L}' |\, \mathfrak{F}(j, a)\rangle_0 \,. \quad (3.16)
\end{aligned}
$$

The general solution (3.16) describes processes in the two-fermion sector at which leptons as well as quarks interact by exchanging electro-weak as well as strong bosons with a mixture of both exchange types in any order. Thus it corresponds in its range of application to gauge theories with grand unification. A comparison will be given in the following section. Higher fermion sectors which include the formation of bosons or of fermions as quark aggregates can be treated in a similar way. However, the calculational effort increases considerably. Finally it should be remarked that the results for the lowest order universal fermion-fermion scattering of III can easily be reproduced with (3.16).

4. Conservation Laws and Unitarization

Provided that the vertex operator $V_{\alpha\beta\gamma\delta}$ is chosen in an appropriate way, the basic field equation is forminvariant with respect to Poincaré-transformations and with respect to an U_1-gauge of the first kind of phase changes of the spinor field operator. While the Poincaré-forminvariance of (1.1) leads to the conservation laws of fourmomentum, mass and spin of the total system, the gauge group admits the definition of a conserved fermion quantum number, which allows the decomposition of the state space into fermion sectors with a definite fermion number. If the full equation for two fermion generations is used as in III, which includes electroweak isospin, then, in addition, a conserved charge quantum number can be defined. As the functional state space reflects the transformation properties of the ordinary Hilbert space of the theory, all conservation laws can be recovered in functional quantum theory, cf. I. If in functional space the projection into the lepton-quark representation is performed, the projection relations (2.15) and (2.16) are compatible with the original U_1-gauge group of $\boldsymbol{\psi}(x)$, but they do not admit a separate definition of a lepton number and a quark number, resp. baryon number. Hence the theory does separately neither contain lepton number conservation nor baryon number conservation, only the total number of baryons and leptons is conserved.

In previous stages of elementary particle theory such a property was considered to be unacceptable. However, the recent development of grand unified gauge theories which was initiated by Pati and Salam [11], has drastically changed the situation.

In these theories it is assumed that the only exact symmetries are gauge symmetries, such as SU_3 color and U_1, which guarantee color- and electromagnetic charge conservation resp. There are no exact gauge symmetries associated with baryon number B or lepton number L conservation, and it is therefore expected that they are violated, cf. Ellis [12]. We therefore assume that the absence of a separate baryon number and lepton number conservation in functional quantum theory of the nonlinear spinor field is not a serious drawback of this approach which necessitates its rejection. On the other hand, in grand unified gauge theories, lepton-baryon conversion processes cannot take place in an unlimited way. Such processes rather underlie certain restrictions. We will at least qualitatively demonstrate that functional quantum theory of the nonlinear spinor field produces similar results. Due to quark confinement a lot of lepton-baryon conversion processes is excluded, which at least partially simulates a separate lepton, resp. baryon, conservation law. In functional quantum theory of the nonlinear spinor field quark-confinement is achieved by unitarization. Hence in order to study approximate lepton, resp. baryon, conservation we have to study unitarization. A correct relativistic invariant unitarization method was given by Stumpf and Scheerer [13]. We qualitatively discuss its application to the two-fermion sector and draw conclusions with respect to selection rules for lepton-baryon conversion processes.

a) We sonsider lepton-lepton scattering processes by assuming the initial state functional to be given by

$$| \mathfrak{F}(s_1 s_2)\rangle_0 = \langle 0 | : \boldsymbol{l}(s_1)\,\boldsymbol{l}(s_2) : | l, k_1, l, k_2 \rangle | D_0 \rangle \quad (4.1)$$

i.e. a configuration of two ingoing leptons. Due to the dynamical law, i.e. the scattering functional equation (3.10), resp. (3.15), lepton-quark conversion processes take place during the scattering process, i.e. in the scattering functional calculation, provided that the kinematical conditions are satisfied. According to the unitarization procedure, such processes can be suppressed in the final result of the scattering calculation by the addition of unobservable ghost configurations

$$| \mathfrak{F}(s_1 s_2)\rangle_0^{\text{ghost}} = a_i \langle 0 | : \boldsymbol{l}(s_1)\,\boldsymbol{A}_0(s_2) : | l, k_1, g, i \rangle | D_0$$
$$+ a_{ij} \langle 0 | : \boldsymbol{A}_0(s_1)\,\boldsymbol{A}_0(s_2) : | g, i, g, j \rangle | D_0 \rangle . \quad (4.2)$$

Hence the addition of such ghost configurations produces for lepton-lepton scattering the conservation of the lepton number.

b) We consider the two-quark sector. Scattering processes between two quarks are forbidden. Hence we do not study initial conditions with ingoing quark configurations. The suppression of forbidden configurations in the initial, resp. boundary conditions is not a matter of the dynamical law. Rather the selfconsistent treatment of quark confinement requires this subsidiary condition for the initial configurations. Therefore, only the two-quark bound states remain. These states are represented by the non-local $(q \otimes q)$-boson states of Section 3. These states do not diagonalize the full functional equation. Rather in the bound state interaction representation interaction terms remain, which represent the higher order selfenergy corrections to the non-local bosons. It cannot be excluded that by these terms the stability of the bound boson state is violated, i.e. that such a non-local boson decays into free quark or free lepton configurations, resp. mixed free quark-free lepton configurations. The suppression of any such decay processes can be formulated by conditions imposed on the scalar product of the state in consideration with corresponding outgoing configurations. For the fulfilment of these conditions only the unobservable free $(g \otimes l)$-, $(g \otimes q)$- and $(g \otimes g)$-configurations are available. These admixtures allow only to suppress the decay of the non-local $(q \otimes q)$-boson into free $(q \otimes q)$-states and free $(q \otimes l)$-states, but not to suppress simultaneously the decay into free $(l \otimes l)$-states. For a simultaneous suppression of both kinds of decays the number of conditions exceeds the number of possible admixtures and this cannot be fulfilled. Hence a $(q \otimes q)$-boson decay into leptons cannot be forbidden.

Similar considerations can be performed for higher fermion sectors. Summarizing and generalizing the results we may conclude that the lepton number for leptonic processes may be assumed to be conserved, but that baryon decay processes into the lepton sector cannot be excluded. These conclusions qualitatively coincide with the results of grand unified gauge theories. For instance, Pati and Salam [11] obtain from their model the non-electromagnetic decay of pions and kaons into leptons as well as the proton decay into leptons. Kane and Karl [14] give a list of possible proton and neutron decay modes within a minimal SU_5 gauge theory and Ellis [12] discusses the various elementary gauge boson exchange processes which lead to baryon number as well as fermion number violation. A more detailed discussion of such processes in the functional quantum theory of the spinor field, however, is only possible if numerical calculations are performed. It is the intention of this paper to provide an appropriate framework for such calculations.

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